CP-even neutrino beam §

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ABSTRACT

The best method of measuring CP violating effect in neutrino oscillation experiments is to construct and use CP-even neutrino beam made of an ideal mixture of $\bar{\nu}_e$ and ν_e of monochromatic lines. We describe how to realize this concept by exploiting accelerated unstable $^{164}_{67}Ho$ hydrogen-like heavy ion in a storage ring, whose decay has both electron capture and beta decay with a comparable fraction. How to measure the CP-odd quantity is explained by using the neutrino beam of this kind.

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Measurement of CP violation in the lepton sector, in particular, in the neutrino sector, is one of the most fundamental problems facing physics beyond the standard model. In this note we develop a concept of CP-even neutrino beam, which serves this purpose.

Concept of CP-even beam The ideal neutrino beam for CP measurement would be a mixture of monochramatic ν_e and $\bar{\nu}_e$ beam for which the detector response is symmetrical, producing after the oscillation equal numbers of $\mu^\pm$ if CP is conserved.

Our proposal is to simultaneously use the monochromatic neutrino ν_e of electron capture (EC) and anti-neutrino $\bar{\nu}_e$ of bound beta ($b\beta$) decay from hydrogen-like heavy ions. Such ions do exist, for example ${}^{164}_{67}Ho^{66+}$. It is an extention of the ideas of monochromatic neutrino beam using EC proposed in [1], [2], and beta beam proposed in [3].

The neutral, unstable atom ${}^{164}_{67}Ho$ has a remarkable property of sharing its decay of a half-life $29\,min$ between EC (60%) and the beta decay (40%) of nearly equal Q-values, $Q_{EC} = 986.8keV$ and $Q_\beta = 962.8keV$ [4]. The hydrogen-like ${}^{164}_{67}Ho^{66+}$ has bound beta decay channels in addition to EC and continuum beta ($c\beta$) decay, and thus produce monochromatic ν_e and $\bar{\nu}_e$ beams once accelerated in a storage ring. One can arrange all lines and a part of the continuum neutrino energy to fall into an optimal range for detection: above $110MeV$ of the muon production, but below multi-pion production in detector placed at a distance. We shall argue that this provides an excellent opportunity of precision experiments to determine the CP violation phase δ [5] as well as the mixing angle θ_{13}.

Multi-lines from bound beta decay and EC A possibility of the bound beta decay that produces monochromatic neutrino has been considered theoretically [6], and their dramatic example that becomes possible only for highly ionized atoms has been demonstrated experimentally [7]. Relative strength of different neutrino lines is proportional to the atomic wave function squared at the nucleus. This factor for the s-wave state of the principal quantum number n is $|\psi_{ns}(0)|^2 = (Z/na_B)^3/\pi$ (using for simplicity the solution of the Schrödinger equation) for hydrogen-like atoms of charge Ze. The ratio of the bound to the continuum contribution is given by

$$r_B = \pi \sum_n N_n (\frac{3.7keV Z}{n\, Q_{c\beta}})^3 (\frac{Q_{c\beta} + \Delta_{\rm BE}}{Q_{c\beta}})^2 \, J^{-1}(\frac{m_e}{Q_{c\beta}}) \,, \tag{1}$$

with N_n the multiplicity factor of available levels and $\Delta_{\rm BE}$ the difference of binding energy in neighboring ions. Here $J(x)$ is related to the phase space integral of the continuum contribution, and in the range, $0.15 \sim 0.74$ for $x = 1/2 \sim 2$. When variation of the level difference $Q_{c\beta} + \Delta_{\rm BE}$ with n is small, the fraction of $1s$ contribution is $1/(2\zeta(3) - 1) \sim 0.71$, $2s$ contribution 0.18, and the rest 0.11. This argument suggests that to obtain a large bound beta rate of $r_B > 1$, we must go to higher Z; for instance, with $Q_{c\beta} + \Delta_{\rm BE} = m_e$, $Z > 57$ corresponds to the bound ratio > 1.

Beta and EC neutrino spectrum from Ho^{66+} The neutrino and anti-neutrino energy spectrum from decay of $^{164}_{67}Ho^{66+}$ at rest is shown in Figure 1. Effect of distorted plane wave under the nuclear Coulomb potential is important and included in the form of the Fermi integral.

Table 1 gives their monochromatic energies and the endpoint energy of the continuum, along with their fractional contributions. To compare the line contribution with the differentical continuum spectrum, we divided the line strength by an energy resolution factor ΔE, which is taken $10keV$ in Figure 1. We denoted decay contributions to excited daughter nuclei by * in Figure 1 and Table 1. It is important to use the wave function of the Dirac equation rather than the Schrödinger equation, since the wave function at nucleus is sensitive to the short distance behavior. The total half-life is shortend from

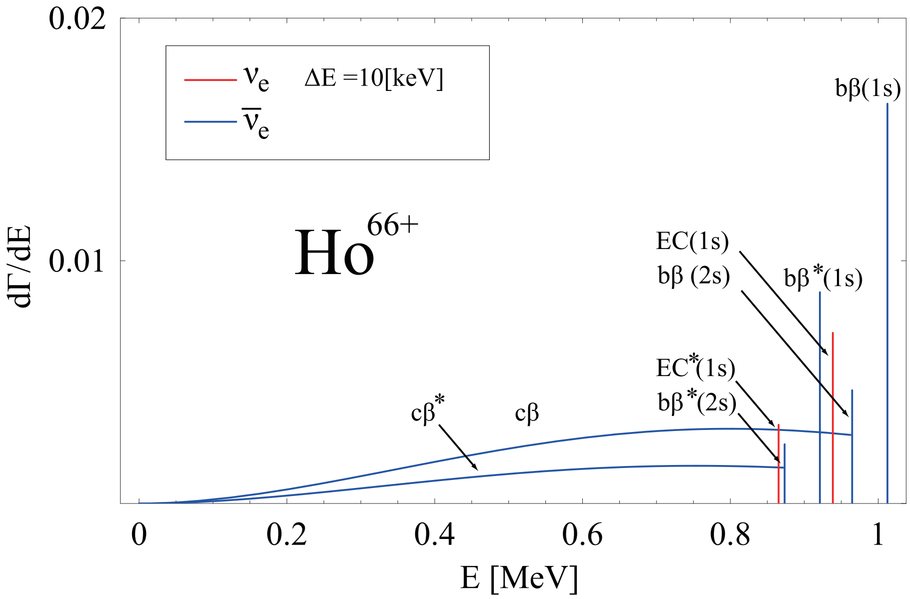

Figure 1: Beta and EC neutrino spectrum from Ho^{66+}

the value of the neutral atom decay by a large factor, to $29min/7.5$, due to two reasons, (1) a larger Fermi integral of stronger Coulomb field, and (2) the appearance of new channels of bound beta decay. Most conspicuous lines from Ho^{66+} have fractions; 0.106 adding bound K- and L- beta decays both to the ground and the first excited nuclear levels, and 0.034 for $EC + EC^*$. On the other hand, for Ho^{65+} which has 2 1s states occupied, the $\bar{\nu}_e/\nu_e$ fraction is 0.024/0.071 for $(b\beta(2s) + b\beta^*(2s))/(EC + EC^*)$. Considering the cross section ratio $(\bar{\nu}_\mu/\nu_\mu) \approx 1/4$ in targets, Ho^{66+} is better suited for the CP-even beam in the energy range of interest (see below).

Table 1@Branching fraction of $c\beta$, $b\beta$, and EC from ${}^{164}_{67}\mathrm{Ho}^{66+}$ decay

decay mode	neutrino energy [MeV]	branching fraction
$c\beta$	0.9628	0.59
$c\beta^*$	0.8714	0.26
$b\beta(1s)$	1.0125	0.054
$b\beta^*(1s)$	0.9211	0.029
$b\beta(2s)$	0.9648	0.015
$b\beta^*(2s)$	0.8734	0.008
EC	0.9386	0.023
EC*	0.8652	0.011

Other candidates of nuclei (^{110}Ag ,^{104}Rh ,^{114}In to the best of our knowledge) of shorter lifetimes ($(30-100)sec$ for neutral atoms) do exist. For instance, He-like ion $^{114}_{49}In^{47+}$ has a lifetime of $\approx 19sec$, and the bound beta fraction 4.2×10^{-3}, and the EC fraction 0.81×10^{-3}. The problem of these isotopes is much separated $b\beta$ and EC energies, different by more than 30%. For $^{114}_{49}In^{47+}$ this separation is $1990/1450 \sim 1.4$.

Sensitivity to θ_{13} and CP parameter, and the best location CP violating effects are present only for the appearance experiment [8], hence we need to detect the muon neutrino from the oscillation, $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$, thus to set the neutrino energy above muon production. The appearance probability for $\nu_e \rightarrow \nu_\mu$ (and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$) is to a good approximation given by

$$P_{\nu_e\nu_\mu(\bar{\nu}_e\bar{\nu}_\mu)} = s_{23}^2\sin^2 2\theta_{13}\,\sin^2\frac{\delta m_{13}^2 L}{4E} + c_{23}^2\sin^2 2\theta_{12}\,\sin^2\frac{\delta m_{12}^2 L}{4E}$$
$$+J\cos\left(\pm\delta - \frac{\delta m_{13}^2 L}{4E}\right)\frac{\delta m_{12}^2 L}{4E}\sin\frac{\delta m_{13}^2 L}{4E}\,, \qquad (2)$$

with $J = c_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}$. We note that for the range of $(\delta m_{12}^2/(4m_{13}^2))^2 < \theta_{13} < \delta m_{12}^2/(4m_{13}^2)$, which implies roughly $\theta_{13} = 0.0025 \sim 0.05$, the last CP sensitive term in eq.(2) is the largest. The matter effect that may mimic CP effects is negligible at low energy of our interest.

To determine θ_{13} and δ with precision, choice of the detector location L is important. Taking into account of the neutrino flux factor $\propto 1/L^2$, 3 terms in the oscillation probability (2) have different $L-$dependence, $\propto L^{-2}\,, L^0\,, L^{-1}$ respectively, when the phase $\varphi = \delta m^2_{13} L/(4E)$ is near the oscillation peak. To maximize simultaneously the flux at the detector and sensitivity to CP parameter δ, the best location is at the first peak $\varphi = \pi/2$. With this choice, the last term in (2) becomes proportional to $\pm J \sin\delta$. Thus the symmetric combination of measured quantities $P_{\nu_e\nu_\mu} + P_{\bar{\nu}_e\bar{\nu}_\mu}$ is sensitive to θ_{13}, while the asymmetric combination $P_{\nu_e\nu_\mu} - P_{\bar{\nu}_e\bar{\nu}_\mu}$ to the CP violation parameter $J\sin\delta$. This choice fixes the relation between E and L to be $L/E \simeq 310[\mathrm{km}]/600[\mathrm{MeV}]$. In Figure 2 we plot the oscillation probability $P(\nu_e \to \nu_\mu)$ as a function of δ for three values of $\sin^2 2\theta_{13} = 0.1\,, 0.05\,, 0.01$.

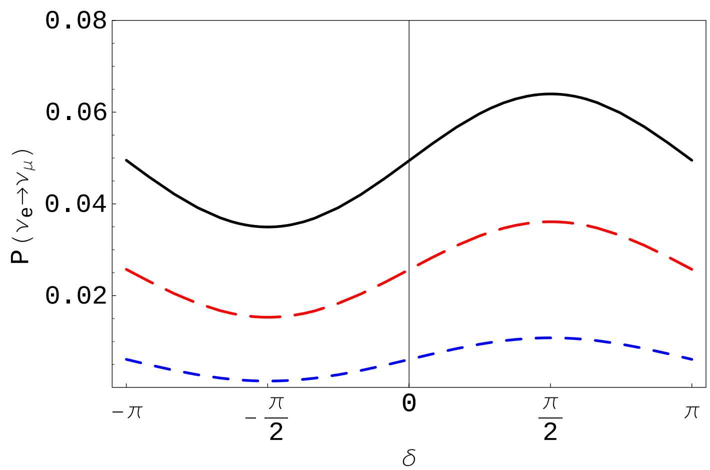

Figure 2: Oscillation probability $P_{\nu_e\nu_\mu}$ at the first peak $\varphi = \pi/2$ for $\sin^2 2\theta_{13} = 0.1\,, 0.05\,, 0.01$ (from top to bottom). $P_{\bar{\nu}_e\bar{\nu}_\mu}$ is obtained with $\delta \to -\delta$.

Optimal choice of beam energy In our proposed scheme, the neutrino energy E can be chosen at will by adjusting the acceleration γ. There are many practical factors to be considered; here we select them mainly from the viewpoint of neutrino detection. The cross section is larger as the energy goes up, but distinction between μ^+ and μ^-, mandatory to establish genuine CP violating effects, becomes harder. Below the single-pion production threshold, backgrounds due to $\pi^\pm$'s are absent; but the cross section is small and the inevitable Fermi motion smears out kinematic relation between $\mu^\pm$ energy and scattering angle. Our optimal choice is 600 MeV, which is above the single-pion but below multi-pion production threshold. At this energy, the cross section for quasi-elastic process ($\nu_\mu n \to \mu^- p$ or $\bar{\nu}_\mu p \to \mu^+ n$) is $3-4$ times as large as for the single-pion production cross section. Other channels such as multi-pion productions are negligible.

A simple Monte Carlo simulation using GEANT4 indicates that a detector consisting of multi-layer magnetized iron and scitillator sandwiches, for example, can cleanly distinguish μ^+ and μ^-, and separate backgrounds from $\pi^\pm$'s using infromation from tracks, range, and energy deposit in the scintillators. Once $\mu^\pm$ identifiction is done, the CP phase δ can be extracted by comparing μ^+ and μ^- yields. Here, the advantage of the monochromatic CP-even beam can be fully exploited; with given γ and δ, one can predict $\mu^\pm$ energy spectrum (and scttering angle correlation) and ratio of $\mu^\pm$ precisely. We note, however, that optimal choice of the neutrino energy may be changed in actual experiments, depending upon chosen technologies of accelerators as well as detectors.

Quantification of CP-eveness The concept of CP-even beam is both detector and neutrino energy dependent, since the ν_μ and $\bar{\nu}_\mu$ cross sections are diffent for different targets, and the line strengths are different at different energies. But one can change the acceleration gamma factor to search for the most sensitive region to the CP measurement for a given multi-line CP-even beam.

One may define the CP-eveness η, using a beam flux $\mathcal{F}(\nu_e)$ from ions weighted by the cross section $\sigma(\nu_\mu)$ at definite neutrino energies;

$$\eta(E\,;\gamma) = \frac{\mathcal{F}(\nu_e)\sigma(\nu_\mu) - \mathcal{F}(\bar{\nu}_e)\sigma(\bar{\nu}_\mu)}{\mathcal{F}(\nu_e)\sigma(\nu_\mu) + \mathcal{F}(\bar{\nu}_e)\sigma(\bar{\nu}_\mu)}\,. \tag{3}$$

Here we assume an iron detector. In the case of ion $^{164}Ho^{66+}$, $\eta \approx 0.17$ selecting the 4 lines above the continuum threshold. The effect of of Fermi motion is included in this estimate. If the continuum beta decay is included, the eveness η can be made much smaller, towards the ideal $\eta \approx 0$.

Rate estimate We first comment on the neutrino energy spread ΔE at detector. Suppose the detector has a lateral size of D; usually D is much smaller than L/γ. Then, the fractional energy spread $\Delta E/E \sim (D\gamma/L)^2$ is negligibly small.

The neutrino flux is of order $\gamma N_{ions}/(\tau L^2)$ at detector, where N_{ions} is the number of decaying heavy ions of lifetime τ. For concreteness, we assume the following numbers for estimate of an event rate; the detector location $L = 310km$, the acceleration factor $\gamma = 300$ boosting the neutrino energy to $E = 600MeV$.

We further assume 10^{14} Ho^{66+} ions, constantly circulating in a storage ring [9], and its 1/3 reach the detector. We take the neutrino cross section of quasi-elastic $\nu_{\mu(e)}N \rightarrow \mu(e)N$ and its anti-neutrino counterpart of $7.7(7.5) \times 10^{-39}cm^2$ and $1.7(1.7) \times 10^{-39}cm^2$, respectively. We include all monochromatic lines of Ho^{66+} given in Table 1. The event rate of $\nu_e + \bar{\nu}_e$ for an iron target is then $O[10]\,(\gamma/300)(N_T/100kt \cdot y)$. Clearly, much more ions are desirable; for instance a new heavy ion candidate of lifetime $\approx 1ms$ enhances the event rate by $\approx 10^5$.

In summary, we discussed the feasibility of multi-line CP-even neutrino beam. Success of precision experiment rests with a large ion factor of N_{ions}/τ.

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